CONDITIONS FOR BOILING START IN CRYOGENIC LIQUIDS

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UDC 532.65:536.423.1

The size of a conical cavity characterizing the start of boiling is estimated on the basis of analyzing the stability of a nucleus of vapor in such a cavity.

A rather important problem in physics of boiling processes is the growth and the breakaway size of bubbles forming on the heater during boiling. This problem is directly involved with determining the dimensions of microcavities, which constitute active centers of boiling.

Usually there appear various kinds of nonuniformities on the heating surface. For the sake of simplicity, we will assume that the surface is "covered" with conical cavities, where half the vertex angle ψ ranges from 0 to 90° and some mean depth h depends on the class of surface finish.

It is well known that the ability of a cavity to reliably retain gas which serves as nucleus of the vaporous phase is the indicator of such a cavity's activity. For cavity to be an active center requires, furthermore, a certain temperature head $\Delta T = T_L - T_s$. The relation between ΔT and the radius r_c of the cavity rim is [1]

$$r_c = 2\sigma T_s/(L\rho''\Delta T). \tag{1}$$

It must be noted here that estimating r_c according to relation (1) is difficult, because of the difficulty of determining T_L. For this reason, one usually deals with the temperature T_H on the heater rather than with the temperature T_L. Experiments [1] have confirmed the validity of such a substitution under conditions of uniform heating of the liquid and its temperature being equal to that of the heater. Under real conditions of heat supply from the heater surface to the liquid, however, the temperature difference $T_H - T_s$ measured in an experiment differs from the quantity $T_L - T_s$ calculated according to relation (1) with r_c known (in [1], these two temperature differences were 11.1 and 1.7°C, respectively). This is attributable to the existence of a boundary layer of liquid, in which the temperature drops from T_H at the heater to T_s at some distance characterizing the thickness of this boundary layer. Taking the thickness of this boundary layer into account [2] makes it possible to correctly determine r_c from the measured T_H temperature. One can apparently assume that in the first approximation r_c and $\delta T = T_H - T_s$ are related through the equality

$$r_c = 2B\sigma T_s/(L\rho''\delta T), \tag{2}$$

with the empirical factor B having any value from 10 to 20 [3].

We will also note that on the left-hand side of equality (1) there should appear r_{0min}, the smallest radius of curvature at the vertex of a bubble which the latter has during its growth. It has been assumed in [1] that the surface of a bubble constitutes a part of a sphere with a radius which varies in time and with a wetting angle θ of 90°, the radius of the sphere becoming minimum when the free surface of the bubble rests on the rim of the cavity so as to form a half-sphere, i.e., when $r_{0min} = r_c$.

We will now determine whether the last equality is valid without the assumptions [1] about the sphericity of the free surface and about $\theta = 90°$. For this purpose we will examine the evolution of the exact shape of a bubble during its slow growth up to breakaway [4-6], beginning from the instant of time when the base of the bubble coincides with the rim of the cavity. We introduce the dimensionless quantity $R_c = r_c b^{1/2}$ with $b = \Delta\rho g/\sigma$. It follows from data on the evolution of the exact shape of a bubble that as soon as $\theta \leqslant 90°$ and $R_c < 0.92$, a bubble resting on the rim of a cavity will intersect a plate at a 90° angle for the first

Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR, Kharkov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 1, pp. 80-85, July, 1983. Original article submitted March 26, 1982.

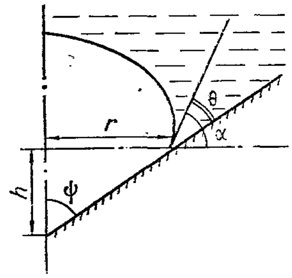

Fig. 1. Schematic diagram of bubble growth in a conical cavity.

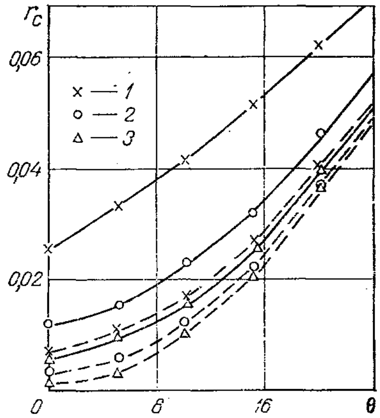

Fig. 2

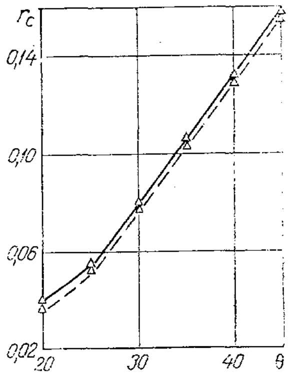

Fig. 3

Fig. 2. Radius r_c (mm) of active cavity at small wetting angles θ (ang. deg): 1) He; 2) H_2O; 3) O_2; solid lines correspond to Class 7 surface finish (GOST 2789-73), dashed lines correspond to Class 10 surface finish.

Fig. 3. Radius of active cavity in water within θ = 20-45° range.

time during its growth. The position of the bubble will then be stable. If $R_c < 0.82$, its radius of curvature at the vertex will then be the minimum one during its entire growth period. The ratio r_c/r_{omin} is 1.14 for $R_c = 0.82$ and, being equal to $(1+\frac{1}{6}R_c^2)$ [7], approaches unity as R_c decreases. Considering that $R_c < 0.82$ for an active vapor nucleating center, one can regard relation (1) as a valid one for $\theta \leqslant 90°$.

It follows from relation (1) that boiling will start earlier (at a smaller ΔT) on a larger cavity if the latter is capable of reliably retaining gas. However, this condition is less likely to be satisfied by large cavities. Therefore, determining the size of an active cavity requires knowing the maximum size of a cavity still capable of retaining gas. We will, accordingly, examine the stability of a gaseous bubble in a conical cavity.

A gaseous bubble deforms during its slow growth in a conical cavity so that its wetting angle θ and thus also angle $\alpha = \pi/2 - \psi + \theta$ (Fig. 1) will remain constant. Loss of stability occurs at the instant when an inflection point appears on the generatrix of the free bubble surface (which coincides with the corresponding equilibrium surface) [8].

We will consider only large angles ψ, since profilograms of solid surfaces [9] indicate that depressions are nearly conical with angles ψ = 80-90°. When ψ is large and θ is small, then α is also small. When α is small, then $R(\equiv rb^{1/2}) = R_*$ at the inflection point can be calculated from the relation $R_* = \frac{1}{4}\left(\frac{3}{2}\right)^{1/2}\sin^2\alpha$ [7]. In the stability-wise critical situation the inflection point is the point of contact with the solid surface, so that, letting

TABLE 1. Theoretical and Experimental Values of Breakaway Radii of Bubbles and of Active Cavities during Boiling of Cryogenic Liquids

Liquid	Surf. finish class, accord. to GOST 2789-73	p, bar	δT, deg	$r_{d\,exp}$, mm	r_c, μm (expression (3))	$r_{d\,theo}$, mm	r_c', μm	$r_{c\,exp}$, μm	$r_{c\,exp}/r_c'$
Oxygen	10	2	3,34	0,15	3,22	0,16	0,40	2,34	5,85
		4	2,85	0,15	3,09	0,16	0,24	2,80	11,67
		6	1,76	0,135	2,98	0,15	0,24	2,26	9,42
		8	1,18	0,12	2,97	0,14	0,24	1,64	6,85
	7	2	1,75	0,26	12,45	0,26	0,76	12,10	15,92
		4	1,40	0,21	12,02	0,24	0,49	7,71	15,73
		6	1,23	0,17	11,80	0,235	0,34	4,52	13,35
		8	0,90	0,145	11,59	0,225	0,31	3,13	10,10
Nitrogen	10	2	2,12	0,15	3,15	0,16	0,35	2,52	7,2
		3	1,57	0,12	3,07	0,155	0,32	1,45	4,53
		4	2,21	0,10	3,05	0,15	0,17	0,96	5,65
	7	2	1,70	0,135	12,5	0,25	0,44	1,84	4,18

$R_c = R_*$, one can calculate the critical radius of a conical cavity as $R_c = \frac{1}{4}\left(\frac{3}{2}\right)^{1/2} \cos^2(\psi - \theta)$. The dimensionless depth $H = hb^{1/2}$ of a critical cavity is determined by the relation $H = R_c \cot\psi$. Upon eliminating the quantity ψ from the relations for R_c and H, we finally obtain

$$\left(\frac{H}{R_c}\cos\theta + \sin\theta\right) = 2\left(\frac{2}{3}\right)^{1/4}\left[R_c\left(1 + \frac{H^2}{R_c^2}\right)\right]^{1/2}. \quad (3)$$

The graphs in Figs. 2, 3 depict the relation $r_c(\theta)$ for helium, oxygen, and water. The surface finish here corresponds to Class 7 and Class 10, respectively. From the known class of surface finish, i.e., known magnitude of R, one can now determine H for a given liquid and from relation (3) determine the limiting radii of active cavities. An analysis of relation (3) suggests that improvement of the surface finish results in a smaller radius of an active cavity and, consequently, a larger temperature difference ΔT for boiling start (according to relation (1)), which has been confirmed in practice. A decrease of θ and σ or an increase of g has a similar effect on r_c. In the latter case (increase of g) r_c must decrease by a factor of $\sqrt{\eta}$, where η is the overload factor.

Sizes of bubbles and active cavities in cryogenic liquids are given in Table 1. The experimental data on breakaway radii r_d of bubbles in the given liquids as well as on heater surface finish, pressures, and temperature drops δT have been taken from another study [10]. In two columns following one another are given theoretical values of r_c (here $\theta = 0°$) based on relation (3), and corresponding values of r_d based on the relation [5]

$$r_d = 1.104\sqrt[3]{r_c/b}. \quad (4)$$

The data in Table 1 indicate that breakaway sizes determined according to the method proposed here represent the upper bound of experimentally determined ones. They are less dependent on the pressure than actually observed in experiments, which can have some effect on the evaluation of dynamic forces. The data in the last three columns can be used for calculating the factor B in expression (2). First $r_c = r_c'$ was calculated according to expression (1), with $\Delta T = \delta T$. Then $r_{c,exp}$ was calculated according to expression (4) and with the use of $r_{d,exp}$. The factor B was calculated as the ratio $B = r_{c,exp}/r_c'$.

We will now examine the law of bubble growth, i.e., the relation $r = \beta\tau^n$. When a bubble grows so that gas flows into it at a constant rate, then the bubble volume is a linear function of time [11] and $n = 1/3$. In real bubbling, however, n has been found to sometimes differ from 1/3, and in the case of boiling — to depend on the pressure [10] with its mean value close to 1/2. This means that gas (vapor) does not flow into a bubble at a constant rate.

Let us examine the bubble growth when the rate of gas flow is controlled by pressure changes in the bubble. Let a bubble form when gas is discharged through a hole of radius r_c.

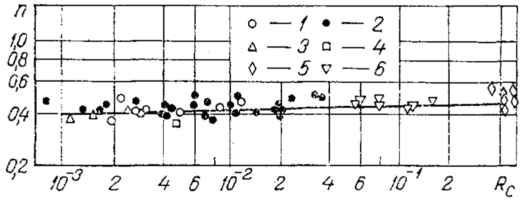

Fig. 4. Dependence of power exponent n in bubble-growth law on radius of active cavity: 1) O_2 boiling [10]; 2) O_2 boiling at $0.02 \leqslant \eta \leqslant 1$ [3]; 3) N_2 boiling [10]; 4) H_2 boiling [15]; 5) H_2O bubbling; 6) N_2 bubbling.

The gas leaves a vessel in which a constant pressure p_1 is maintained. As the bubble slowly grows, the pressure p_2 inside it will vary according to the relation

$$p_2 = \Delta p_0 + \rho' gl + p, \quad l = l_0 - l_1(v). \tag{5}$$

In this case the flow rate Q of gas discharged through the hole can be estimated on the basis of the relation [12]

$$Q = \pi r_c^2 \sqrt{2 \cdot 9.81 \frac{k}{k-1} \rho_1 p_1 \left[\left(\frac{p_2}{p_1} \right)^{2/k} - \left(\frac{p_2}{p_1} \right)^{k+1/k} \right]}. \tag{6}$$

Using the results of another study [4], where Δp_0 and l_1 have been determined as functions of the volume v for $0.02 \leqslant r_c\sqrt{b} \leqslant 0.5$, as well as the relations $\Delta p_0(v)$ and $l_1(v)$ established for $0.001 \leqslant r_c\sqrt{b} \leqslant 0.02$ on the assumption of a spherical bubble, we obtain from relations (5), (6) the relations $Q = f(v)$ with r_c = const. Integrating $Q = dv/d\tau$ yields the volume v as a function of time, and, in turn, this relation determines both the exponent n and the coefficient β. The radius of a bubble can then be expressed through its volume v as $r = \sqrt[3]{3v/4\pi}$.

It is to be noted that the trend of the $\Delta p_0(v)$ curves depends largely on the value of R_c [4]. When R_c is small, then the function $\Delta p_0(v)$ ascends steeply to some maximum and then descends until stability is lost. As R_c increases, the range of increasing Δp_0 widens, and at some sufficiently large R_c the loss of stability occurs before Δp_0 begins to decrease. Calculations have been made for small R_c values ($R_c \leqslant 0.5$) in the range of decreasing pressure. It has been found that n depends on R_c only (β depends on the other parameters) and lies within $0.4 \leqslant n \leqslant 0.45$ for R_c within the 0.001-0.5 range, becoming larger as R_c increases.

Studies of boiling of cryogenic liquids have revealed that an increase of pressure causes a decrease of the exponent n in the quasistatic mode of bubble growth [10]. We will draw an analogy between the growth of a vapor bubble in a cavity of radius r_c and the growth of a gas bubble during bubbling at a hole of the same radius. For this we have to evaluate the pressure dependence of R_c. Using the pressure dependence of the dimensionless complex $\sigma T_s\sqrt{b}/L\rho''$, determined by the properties of the liquid and its vapor, and using the pressure dependence of ΔT determined experimentally for oxygen [10], hydrogen [13], and water [14], we obtain from relation (1) the sought relation for R_c. This relation is $R_c \sim p^{-\gamma}$, with γ = 0.97, 1.41, and 1.75 for hydrogen, oxygen, and water, respectively. One may propose that this experimentally observed decrease of n with increasing pressure is a consequence of the decrease of the radius R_c of an active cavity.

The calculated values of the exponent n were checked experimentally in bubbling tests. The liquids used there were water and liquid nitrogen. The experimental data, along with the results of theoretical calculations, are shown in Fig. 4. On the same diagram are also shown data on boiling of nitrogen and oxygen [10], hydrogen [15], and oxygen at $0.02 \leqslant \eta \leqslant 1$ [3]. These results suggest that the bubble growth during bubbling as well as during boiling in the quasistatic mode is determined by the pressure drop and that the observed pressure dependence of the power exponent n manifests the pressure dependence of the breakaway size of bubbles in the quasistatic mode.

NOTATION

h, H, dimensional and dimensionless depth of a conical cavity; ψ, half the vertex angle of a conical cavity; r_c, R_c, dimensional and dimensionless radius of the cavity base; r_{0min}, minimum radius of curvature at the cavity vertex; R, dimensionless radius of a bubble; R_*, dimensionless critical radius of a conical cavity; r_d, breakaway radius of a bubble; $r_{d,exp}$, experimentally determined breakaway radius of a bubble; r_c', radius of a cavity according to relation (1); $r_{c,exp}$, radius of a cavity determined from $r_{d,exp}$; T_L, mean temperature of the liquid at the bubble surface; T_s, saturation temperature; T_H, temperature at the heater; $\Delta T = T_L - T_s$; $\delta T = T_H - T_s$; θ, wetting angle; σ, coefficient of surface tension; L, latent heat of evaporation; ρ'', density of the gas (vapor); Δρ, difference between density of liquid and density of vapor; ρ_1, density of the gas under pressure p_1; g, gravitational acceleration; α, angle of inclination (to the horizontal) of a tangent to the bubble surface; η, overload factor; β, B, coefficients; τ, time; n, γ, power exponents; Δp_0, magnitude of the Laplace pressure at the tip of a bubble; p, gas pressure over the liquid surface; p_1, pressure inside the vessel from gas flows through bubbling hole; p_2, pressure in a bubble; l, height of the liquid column above a bubble; l_0, level of the liquid in the vessel; l_1, height of a bubble; v, volume of a bubble; Q, gas flow rate; $k = C_p/C_v$, adiabatic exponent; C_p, isobaric specific heat; C_v, isochoric specific heat.

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